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Estimation of the bi-modulus of materials through deformation measurement in a Brazilian disk test

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ABSTRACT

A completely analytical experimental theory is developed to determine the elastic tensile modulus E_t and the elastic compressive modulus E_c of materials simultaneously through the deformation and displacement measurement in the Brazilian disk loaded by a pair of radial concentrated forces. Based on the proposed test theory, there are totally four kinds of estimation methods proposed. Two groups of experimental data are used to validate the proposed test theory. The analysis indicates that the estimated E_t and E_c of materials have high reliability when measured horizontal and vertical displacements on the diameters are used simultaneously. The measurement error of displacement results in that the estimated E_t and E_c of materials follow the *t* location-scale distribution very well. The validation works indicate that the proposed test method is feasible and liable, and is convenient to determine the E_t and E_c of materials simultaneously through the simple indirect tension test (the Brazilian disk test).

1. Introduction

It is well known that the mechanical behavior of rock materials under tensile and compressive stress is significantly different, due to the existence of macro- or micro-cracks in rock mass. The failure of engineering material is much easier to occur under tensile stress than that under compressive stress. In most previous investigations, the differences between the tensile and compressive behavior of rock materials are not considered adequately. Generally, the compressive parameters, such as compressive elastic modulus E_c determined from the uniaxial compressive test, are used to evaluate the stress and displacement field of engineering rock masses, such as slopes, tunnels, and large carves in mountains, regardless of whether the rock is in the tensile or compressive regime. This method will undoubtedly underestimate the magnitude of the displacement field. Consequently, a high risk of failure would be raised. Therefore, it is important for rock engineers to use the suitable parameters for the zones in rock mass which is in the tensile state. Development of an effective and feasible method to determine the tensile parameters, especially the elastic tensile modulus E_t is necessary and meaningful.

In recent years, many researchers have paid attention to the problem how to determine the elastic modulus of rock materials under tensile stress. Generally, two methods are used. The first one is the direct tension test. The second method is the indirect tension test, mainly the Brazilian disk test. Due to the complexity of operation and the stress concentration at the end of samples, the direct tension test is usually not adopted. The Brazilian disk test is a widely used method. Ye et al. [1] developed an analytical method to approximately determine the elastic tensile modulus E_t adopting a pair of concentrated forces radially applying on a Brazilian disk. In this test method, a strain gage is glued horizontally on the central point to record the strain in the region the strain gage pasted, rather than the strain of the central point. The priority of this method is that the effect of the length of strain gage used on the magnitude of measured strain is considered, and the theoretical expression to determine the elastic tensile modulus E_t from experimental data is completely analytical. Therefore, the results of elastic tensile modulus E_t is more reliable than that determined by Liu [2], who only considered the deformation of the central point of the disk. Recently, based on the analytical theory proposed by Ye et al. [1], Gong et al. [3] developed a method to determine the elastic tensile modulus through monitoring displacements between two points located on the circular disk. However, the expression adopted to determine E_t is not a general analytical solution. Liu [4] proposed a method to determine the elastic parameters (E and ν) through monitoring the

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whole deformation field on the disk using the DIC technique [5] when the Brazilian disk is applied by two rigid flat plates. In this method, the difference of tensile and compressive behavior of materials is completely not considered. The elastic tensile modulus is considered the same with the elastic compressive modulus. Obviously, this assumption is inappropriate.

In this study, an experimental method to determine the elastic tensile modulus E_t, and compressive modulus E_c simultaneously through measuring the displacement field on the Brazilian disk is developed. In this method, a series of complete analytical expressions are proposed; and the differences of tensile and compressive behaviors are adequately considered. Finally, this proposed method is validated by two groups of experimental data reported in previous literatures. The probabilistic analysis and the sensitivity analysis are performed to investigate the reliability of the proposed test theory. The priorities of this proposed test method to the previous method proposed by Ye et al. [1] include the following: (1) the measurement of displacement is much more convenient than the stain measurement using the strain gauges in tests; (2) the elastic compressive modulus E_c can be determined simultaneously with the elastic tensile modulus E_t; (3) once the displacement distribution on the disk is measured at a moment on the same samples, a series of E_t and E_c of materials can be estimated using the proposed method. The probabilistic analysis can be performed for the data of estimated E_t and E_c, which makes the estimated bi-modulus of materials have better reliability.

2. Test theory

In this study, the indirect tension test, the Brazilian disk test is adopted to determine the elastic tensile and compressive modulus. From the point of view of physics, the analytical solution of stresses field in the Brazilian disk considering the difference of elastic tensile modulus E_t and compressive modulus E_c should be adopted. It means that the elastic tensile modulus E_t should be used in the directions where the tensile stress is applying. If the applied stress is compressive in the other directions on the same micro-volume, the compressive modulus E_c should be used in these directions. However, this kind of analytical solution is not available so far. The reason is that there are two moduli E_t and E_c involved in the analysis; and we cannot find a general method which could judge that the stress is tensile or compressive in its direction. The numerical methods may be the best tools to accurately determine the stress and displacement field considering the differences of E_t and E_c for rock materials. Some researchers [6] think that the analytical solutions for the Brazilian disk test for transversely anisotropic materials [7,8] are equivalent with the analytical solution considering the differences of E_t and E_c. This assumption is debatable. After all the concept of the differences between the E_t and E_c of rock materials is completely different with the transversely anisotropy of materials. However, the E_t and E_c also would significantly differ for isotropic rock materials, for example the pure white marble. In this study, for the simplicity and convenience, the Brazilian disk is treated as isotropic and homogeneous material. Ye et al. [9] derived the analytical solution of the stresses field in the Brazilian disk under a pair of radial concentrative force based on the Airy stress function method. Although the final expressions of the stress field are the same with that proposed by Muskhelishvili [10], the derivation process is completely different. The reason why the concentrative force is applied, rather than a distributive force, is that it is difficult to know the contract angle between the loading curve and the disk [1]. The test method adopted in this study is that the stress is applied by a pair of concentrated forces (Fig. 1); meanwhile, the displacements of some points in disk are monitored adopting the DIC technique or the displacement sensors. Finally, the elastic tensile and compressive moduli of materials are calculated from the obtained displacement data by using the analytical expressions proposed in the following part.

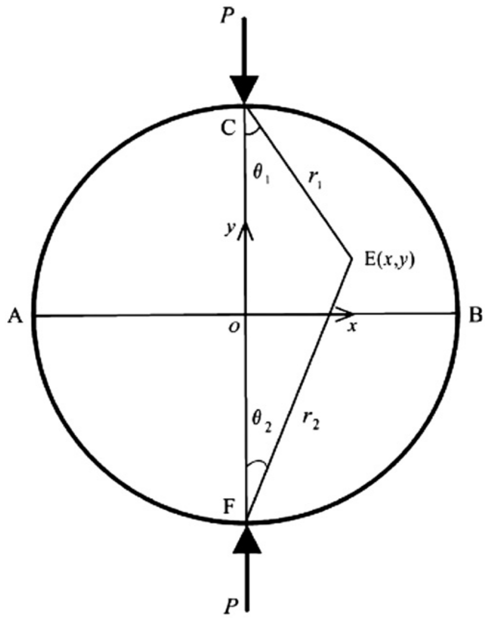

Fig. 1. Loading state of the Brazilian disk test.

The analytical solution of stresses field in an isotropic Brazilian disk under concentrative forces is given as [9,10]

$$
\begin{aligned}
\sigma_x &= \frac{2P}{\pi l}\left\{\frac{((D/2)-y)x^2}{(((D/2)-y)^2+x^2)^2}+\frac{((D/2)+y)x^2}{(((D/2)+y)^2+x^2)^2}-\frac{1}{D}\right\}\\
\sigma_y &= \frac{2P}{\pi l}\left\{\frac{((D/2)-y)^3}{(((D/2)-y)^2+x^2)^2}+\frac{((D/2)+y)^3}{(((D/2)+y)^2+x^2)^2}-\frac{1}{D}\right\}\\
\tau_{xy} &= \frac{2P}{\pi l}\left\{-\frac{((D/2)-y)^2}{(((D/2)-y)^2+x^2)^2}+\frac{((D/2)-y)^2}{(((D/2)+y)^2+x^2)^2}\right\}
\end{aligned}
\tag{1}
$$

where P is the applied force, l is the thickness of disk, D is the diameter of disk. On the diameter AB and CF, the stresses are

$$
\begin{aligned}
\sigma_x &= \frac{2P}{\pi Dl}\left\{\frac{16D^2x^2}{(D^2+4x^2)^2}-1\right\}\\
\sigma_y &= \frac{2P}{\pi Dl}\left\{\frac{4D^4}{(D^2+4x^2)^2}-1\right\} \quad \text{on } AB\\
\tau_{xy} &= 0
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
\sigma_x &= -\frac{2P}{\pi Dl}\\
\sigma_y &= \frac{2P}{\pi Dl}\left\{\frac{4D^2}{D^2-4y^2}-1\right\} \quad \text{on } CF\\
\tau_{xy} &= 0
\end{aligned}
\tag{3}
$$

The horizontal displacement $u(x,t)$ on diameter AB can be determined by

$$
u(x,t) = \int_0^x \left(-\frac{\sigma_x}{E_t}+v\frac{\sigma_y}{E_c}\right)dx \tag{4}
$$

Similarly, the vertical displacement $v(y,t)$ on diameter CF can be determined by

$$
v(y,t) = \int_0^y \left(\frac{\sigma_y}{E_c}-v\frac{\sigma_x}{E_t}\right)dy \tag{5}
$$

where the E_t and E_c are the elastic tensile and compressive modulus of rock materials. v is Poisson's ratio. Substituting Eq. (2) into Eqs. (4) and (5), and applying following two integral formulations yields

$$\int_0^x \frac{16x^2}{(4x^2+D^2)^2}dx = \frac{1}{D}\arctan\frac{2x}{D} - \frac{2x}{4x^2+D^2}$$
$$\int_0^x \frac{4}{(4x^2+D^2)^2}dx = \frac{1}{D^3}\arctan\frac{2x}{D} + \frac{2x}{D^2(4x^2+D^2)}$$
$$\int_0^y \frac{1}{D^2-4y^2}dy = \frac{1}{2D}\ln\frac{D+2y}{D-2y} \tag{6}$$

The horizontal displacement on the diameter AB can be expressed as

$$u(x,t) = \frac{2P}{\pi l}\left\{\left(\frac{x}{D} - \arctan\frac{2x}{D}\right)\left(\frac{1}{E_t} - \frac{v}{E_c}\right) + \frac{2Dx}{4x^2+D^2}\left(\frac{1}{E_t} + \frac{v}{E_c}\right)\right\} \tag{7}$$

and the vertical displacement on the diameter CF can be expressed as

$$v(y,t) = \frac{2P}{\pi l}\left\{\frac{2}{E_c}\ln\frac{D+2y}{D-2y} + \frac{y}{D}\left(\frac{v}{E_t} - \frac{1}{E_c}\right)\right\} \tag{8}$$

It can be easily seen that the bi-modulus of materials E_t and E_c is not explicitly expressed in Eqs. (7) and (8). If the horizontal and/or vertical displacements at some points on the diameter AB and/or CF of the Brazilian disk are measured in tests, then the elastic tensile and compressive modulus of materials can be determined simultaneously through solving the equations consisting of Eq. (7) and/or Eq. (8). Totally, there are three types of method that applying Eqs. (7) and (8) to determine the elastic tensile and compressive modulus of materials in application. If only the horizontal displacements of at least two points on the diameter AB are measured, Eq. (7) can be used to determine the bi-modulus of the materials (labelled as method A). If only the vertical displacements of at least two points on the diameter CF are measured, Eq. (8) can be used (labelled as method B). The last method involves that if both the horizontal displacements of points on the diameter AB, and the vertical displacements of points on the diameter CF are measured, Eqs. (7) and (8) can be used simultaneously (labelled as method C) to form the equations for E_t and E_c. Another characteristic of Eqs. (7) and (8) is that they are both odd symmetry. This characteristic requires that we cannot adopt the horizontal displacements of two symmetric points on AB and Eq. (7), or the vertical displacements of two symmetric points on the CF and Eq. (8) to determine the bi-modulus of materials, since the equations for the E_t and E_c are not positive definite if the horizontal displacements of symmetric points are used in Eq. (7), or the vertical displacements of symmetric points are used in Eq. (8).

From Eq. (7), the horizontal displacement at the ends of the diameter AB ($x=-R$ or R) can be written as

$$u(x,t) = \frac{2P}{\pi l}\left\{\left(\frac{1}{E_t} - \frac{\pi}{4}\frac{1}{E_t}\right) + \frac{\pi}{4}\frac{v}{E_c}\right\} \tag{9}$$

In Eq. (8), if the $y=R$ or $-R$, the vertical displacements of the ends of diameter CF is infinite. It is indicated that the analytical solution of the vertical displacement in the Brazilian disk under a concentrated force loading is singular at the applying point. This phenomenon reminds us that the displacement measured at the points which are very near to the two applying points should not be used to determine the bi-modulus of materials adopting the proposed methods in this study.

In the above proposed test theory, the difference between the E_t and E_c of materials is taken into consideration. In order to determine the bi-modulus of materials simultaneously, the displacements of at least two points on the diameter AB and/or CF are required to be measured simultaneously in a Brazilian disk test. However, only the displacement at one point on the diameter AB or CF is available in some cases. In order to approximately estimate the elastic tensile or compressive modulus of materials, a simplification that does not care about the difference between the E_t and E_c is necessary. Keeping this simplification in mind, and considering that the deformation on the diameter AB and CF is dominated by E_t and E_c respectively, Eqs. (7)–(9) can be written as

$$u(x,t) = \frac{2p}{E_t \pi l}\left\{(v-1)\arctan\frac{2x}{D} + \frac{2Dx}{4x^2+D^2}(1+v) + \frac{x}{D}(1-v)\right\} \tag{10}$$

$$v(y,t) = \frac{4p}{E_c \pi l}\ln\frac{D+2y}{D-2y} + \frac{2p}{E_c \pi l}(v-1)\frac{y}{D} \tag{11}$$

$$u(x,t) = \frac{p}{E_t l}\left\{\frac{1}{2}v - \frac{1}{2} + \frac{2}{\pi}\right\} \tag{12}$$

Under this simplification, the bi-modulus of materials E_t and E_c can be explicitly expressed by the horizontal displacement on the diameter AB and the vertical displacement on the diameter CF. The bi-modulus of materials can be approximately estimated adopting Eqs. (10) and (11) (labelled as method F). The error brought by this simplification will be discussed below.

3. Validation with experimental data

In this section, the above proposed test method and the analytical expressions for determining the E_t and E_c of materials are validated to show the feasibility of this test theory. Two groups of experimental data provided by Liu [4] and Gong et al. [3] are adopted to perform the validation work.

3.1. Liu's experiment

Liu [4] conducted a Brazilian disk test taking the epoxy resin as the material. In this test, the loading plate is flat. At the later phase of loading, the contact area between the flat and the resin disk is large so that the force acting on the disk is not the concentrative force, but a distributive force. However, at the beginning stage, the force acting on the disk could approximately be considered as a concentrative force due to the small contact area between the disk and the flat plates. Liu [4] monitored the variation process of the displacement field in the disk using the DIC technique during the test. Here, only the displacements at one moment in the early stage are adopted to validate the proposed test theory. The distribution of horizontal displacement $u(x)$ on the diameter AB and the vertical displacement $v(y)$ on the diameter CF when $P/lR=36.1$ MPa is shown in Fig. 2. Due to the fact that the force acting on the disk $P/lR=36.1$ MPa at this moment is less than one-fourth of the yielding force $P/lR=160$ MPa, the deformation in the Brazilian disk at this moment is nearly perfectly elastic. Therefore, the bi-modulus of materials determined by the displacement field in the disk at this motion is elastic modulus of materials. In Fig. 2, it is found that the horizontal displacement on AB and the vertical displacement on CF are both odd symmetry; and the horizontal and vertical displacement at the center of disk are both 0. The diameter of resin disk $D=16.84$ mm, thickness $l=6.75$ mm. Poisson's ratio v is 0.364 [4].

Firstly, the method A is adopted to determine the bi-modulus of the epoxy resin based on the horizontal displacements on the diameter AB. The horizontal displacements of any two different points on AB are chosen, and substituted into Eq. (7) to form the nonlinear equations for the unknown E_t and E_c. The bi-modulus of epoxy resin can be easily determined by solving the nonlinear equations. In calculation, the horizontal displacements of any two different asymmetric points on AB are randomly selected from the

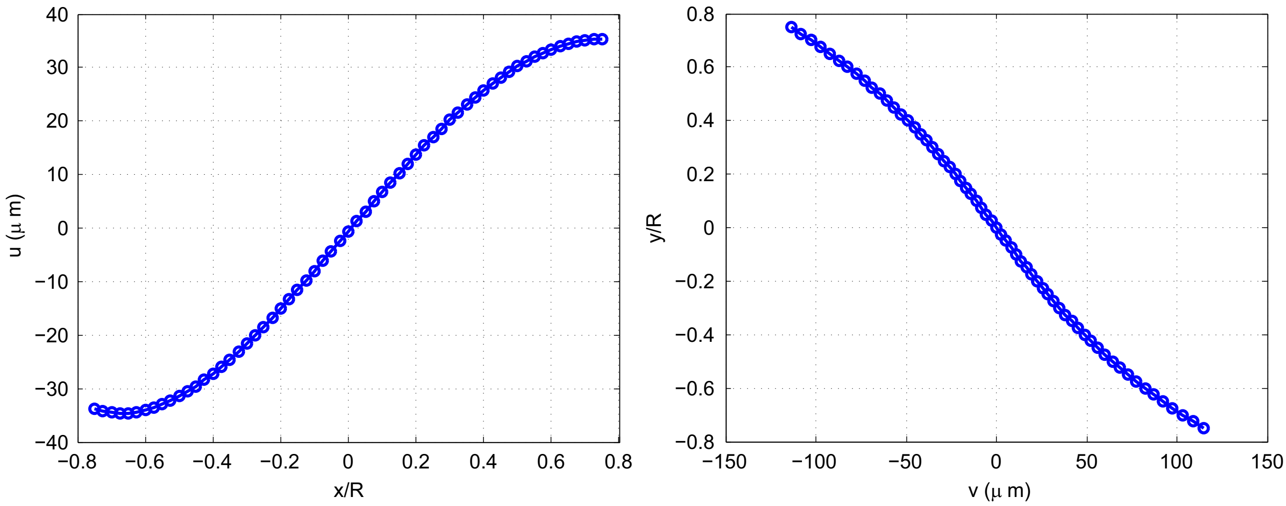

Fig. 2. Distribution of $u(x)$, $v(y)$ on the diameter *AB* and *CF* in the Brazilian disk when $P/lR=36.1$ MPa.

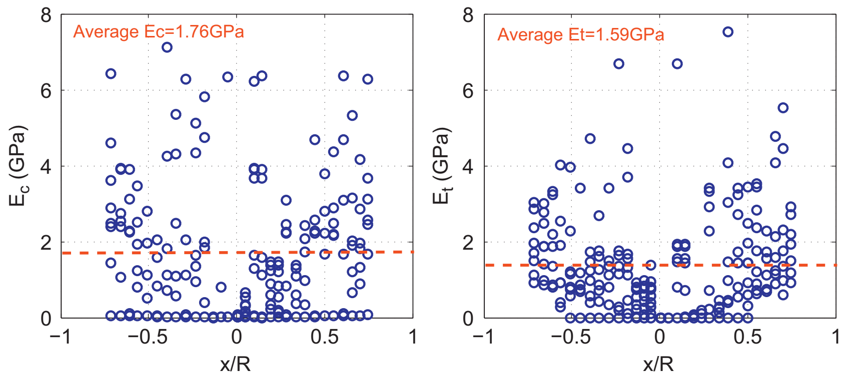

Fig. 3. Bi-modulus of the epoxy resin at the moment when $P/lR=36.1$ MPa determined by adopting the horizontal displacements on the diameter *AB* and Eq. (7) (method A).

all horizontal displacement data obtained in the Brazilian disk test. A series of the elastic tensile and compressive modulus are determined based on the different combination of the horizontal displacements data.

Fig. 3 shows the determined bi-modulus of the epoxy resin at the moment when $P/lR=36.1$ MPa. The average elastic tensile and compressive modulus of the epoxy resin are estimated as 1.59 GPa and 1.76 GPa respectively. In Fig. 3, it is found that the elastic tensile and compressive modulus of the epoxy resin determined by method A is significantly scattered due to the measurement error of displacement. There is no area where the E_t or E_c is obviously gathered.

The probability analysis is performed for all the data of the elastic tensile and compressive modulus of the epoxy resin determined by method A. Fig. 4 demonstrates the histogram and the fitted probability density function for the E_t and E_c determined by method A. As demonstrated in Fig. 4, the elastic compressive modulus E_c determined by method A approximately follows the Nakagami distribution. The weighted mean value of E_c is 1.64 GPa, which is near to the arithmetic mean value 1.76 GPa shown in Fig. 3. From the histogram of E_c in Fig. 4, it is observed that a great number of E_c determined by method A are near to zero. This is not reasonable. It would be attributed to that the elastic compressive modulus of materials is very sensitive to the horizontal displacement of points on *AB* if method A is used. This kind of sensitivity analysis will be performed later. It results in that the small measurement error on the horizontal displacement could bring huge error on the E_c. However, the trend observed in the histogram of E_t in Fig. 4 is significantly different with that of E_c. It is found that the elastic tensile modulus E_t approximately follows the *t* location-scale distribution. The weighted mean value is 1.29 GPa. The difference between the weighted mean value and the arithmetic mean value is 0.3 GPa.

Secondly, method B is adopted to determine the bi-modulus of the epoxy resin based on the vertical displacements on the diameter *CF*. The vertical displacements of any two different points on *CF* are chosen, and substituted into Eq. (8) to form the nonlinear equations for the unknown E_t and E_c. The bi-modulus of epoxy resin can be determined by solving the formed nonlinear equations. Similarly, in calculation, the vertical displacements of any two different asymmetric points on *CF* are randomly selected from the all vertical displacement data obtained in the Brazilian disk test. A series of the elastic tensile and compressive modulus are also determined based on the different combination of the vertical displacements data.

Fig. 5 shows the bi-modulus of the epoxy resin at the moment when $P/lR=36.1$ MPa determined by using method B based on the measured vertical displacement on the diameter *CF*. The average elastic tensile and compressive modulus of the epoxy resin at the moment when $P/lR=36.1$ MPa is 1.63 GPa and 5.59 GPa respectively. In Fig. 5, it can be seen that most of the estimated E_c is obviously gathered in the range of 4–8 GPa, and most of the estimated E_t is less than 2 GPa. Fig. 6 demonstrates the histogram and the fitted probability density function for the E_t and E_c determined by method B. From Fig. 6, it can be seen that the elastic compressive modulus E_c determined by method B follows the *t* location-scale distribution very well. The weighted mean value is 5.98 GPa, which is very near

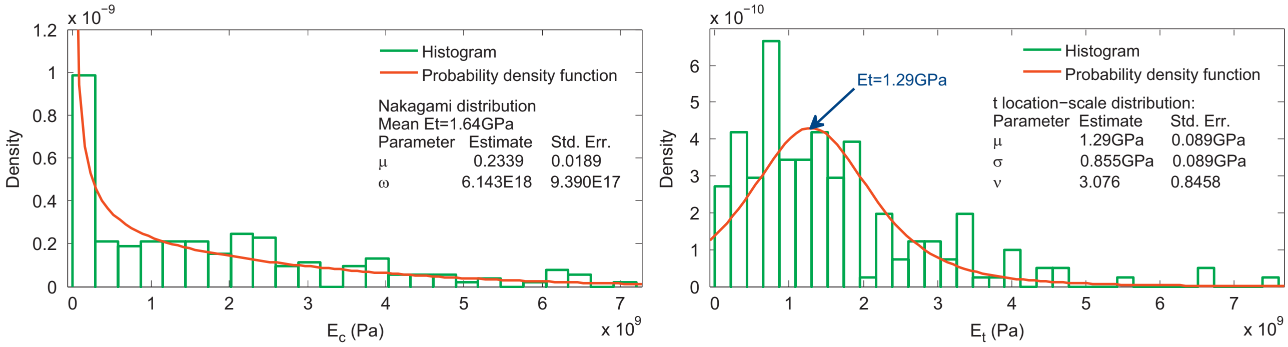

Fig. 4. Histogram and the fitted probability density function for the data of bi-modulus of epoxy resin in Fig. 3.

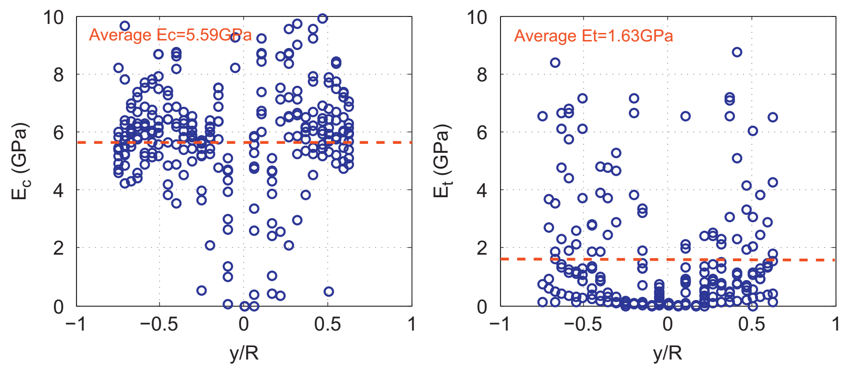

Fig. 5. Bi-modulus of the epoxy resin at the moment when $P/lR = 36.1$ MPa determined by adopting the vertical displacements on the diameter *CF* and Eq. (8) (method B).

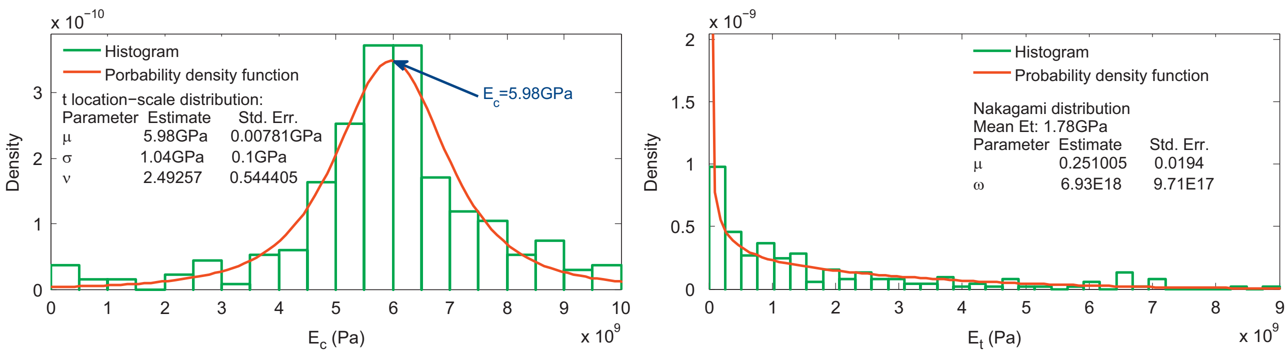

Fig. 6. Histogram and the fitted probability density function for the data of bi-modulus of epoxy resin in Fig. 5.

to the arithmetic mean value 5.59 GPa. The good agreement between the histogram and the fitted probability density function indicates that the elastic compressive modulus E_c determined by method B based on the vertical displacement on the diameter *CF* is reliable. In Fig. 6, the distribution of elastic tensile modulus E_t is also found to approximately follow the Nakagami distribution. As the distribution of E_c in Fig. 4, there are also a great number of E_t determined by method B are near to zero. This is also not reasonable. Therefore, we should be prudential when adopting method B to determine the elastic tensile modulus E_t of materials.

Thirdly, method C is adopted to determine the bi-modulus of the epoxy resin based on the horizontal on *AB* and vertical displacements on *CF*. The horizontal displacement of one point on *AB*, and the vertical displacements of one points on *CF* are randomly chosen, and substituted into Eqs. (7) and (8) to form the nonlinear equations for the unknown E_t and E_c. Similarly, a series of the elastic tensile and compressive modulus are determined based on the different combination of the horizontal and vertical displacements data.

Fig. 7 shows the bi-modulus of the epoxy resin at the moment when $P/lR = 36.1$ MPa determined by using method C based on the measured horizontal on *AB* and the vertical displacement on *CF*. From Fig. 7, it is found that the elastic tensile and compressive modulus of the epoxy resin determined by method C both has good consistency. The E_c is mainly in the range 6–8 GPa, and the E_t is mainly in the range of 1.3–1.6 GPa. This phenomenon observed from Fig. 7 indicates that the method C is highly reliable to determine the bi-modulus of materials in the Brazilian disk test. The arithmetic mean value of E_t and E_c is 1.5 GPa and 6.83 GPa respectively.

Fig. 8 demonstrates the histogram and the fitted probability density function for E_t and E_c determined by method C. As illustrated in Fig. 8, the estimated E_t and E_c both follow the *t* location-scale distribution very well. The weighted mean value of E_t and E_c is 1.474 GPa and 7.188 GPa respectively. The good consistency shown in Fig. 7, and the good agreement with *t* location-scale distribution shown in Fig. 8 indicate that method C is much better than methods A and B to determine the

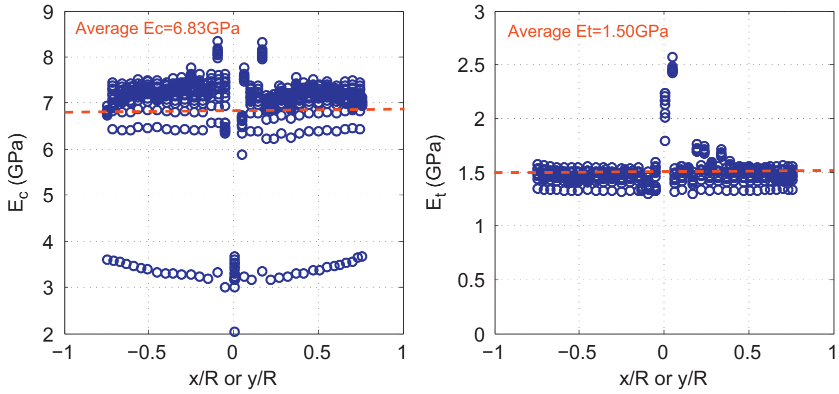

Fig. 7. Bi-modulus of the epoxy resin at the moment when $P/lR=36.1$ MPa determined by adopting the horizontal and vertical displacements on the diameter *AB* and *CF*, and Eqs. (7) and (8) (method C).

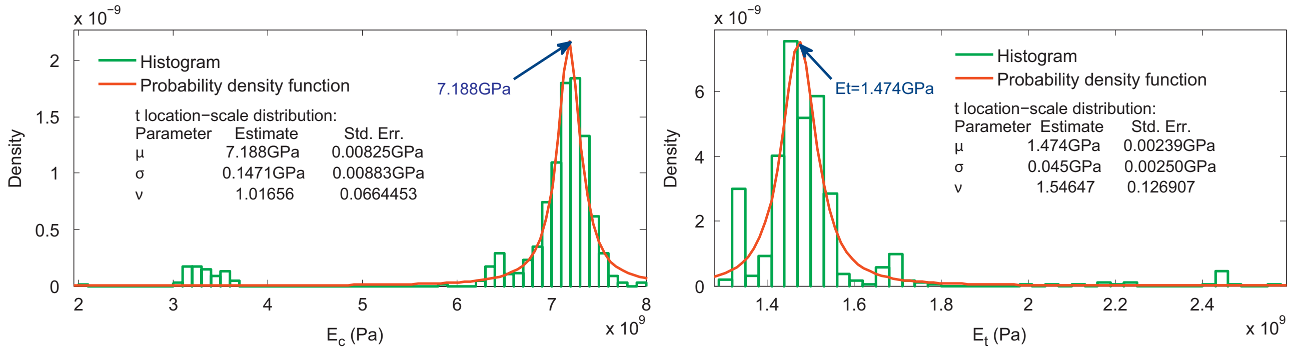

Fig. 8. Histogram and the fitted probability density function for the data of bi-modulus of epoxy resin in Fig. 7.

bi-modulus of materials. In summary, the bi-modulus of epoxy resin determined by method C have the best reliability comparing methods A and B. Finally, the elastic tensile modulus of the epoxy resin is estimated as about 1.5 GPa, the elastic compressive modulus of the epoxy resin is estimated as 6.8–7.2 GPa. The elastic tensile modulus E_t is much less than that of the elastic compressive modulus for the epoxy resin at the moment when the $P/lR=36.1$ MPa. Based on this estimation, it is found that the elastic tensile modulus of materials estimated by method A, and the elastic compressive modulus of materials estimated by method B is also acceptable to some extent.

As mentioned above, the elastic compressive modulus E_c of the epoxy resin determined by method A, and the elastic tensile modulus E_t of the epoxy resin determined by method B both follow the Nakagami distribution, in which the frequency of E_t and E_c near to zero is relatively high. This phenomenon is not reasonable to some extent. The error for the E_t and E_c estimation brought by the measurement error in tests is relatively huge, and cannot be acceptable. This is attributed to the high sensitivity of E_c to the horizontal displacement in method A, and of E_t to the vertical displacement in method B. If the elastic tensile and compressive modulus E_t and E_c are very sensitive to the displacement, then the small measurement error of displacement will bring unacceptable results.

Figs. 9 and 10 illustrate the sensitivity of the bi-modulus of the epoxy resin to the horizontal and vertical displacement based on Eqs. (7) and (8). The sensitivity of the bi-modulus is defined as

$$k=\left|\frac{E_c}{u}\right| \quad \text{or} \quad k=\left|\frac{E_c}{v}\right| \quad \text{or} \quad k=\left|\frac{E_t}{u}\right| \quad \text{or} \quad k=\left|\frac{E_t}{v}\right| \qquad (13)$$

As illustrated in Figs. 9 and 10, the sensitivity of the elastic compressive modulus E_c to the horizontal displacement is about 6–10 times of the sensitivity of the elastic tensile modulus E_t to the horizontal displacement on *AB*. While, the sensitivity of the elastic tensile modulus E_t to the vertical displacement is about 1.33 times of the sensitivity of the elastic compressive modulus E_c to the vertical displacement on *CF*. Due to the high sensitivity, it is not suggested to estimate the elastic compressive modulus E_c of materials using method A, and to estimate the elastic tensile modulus E_t of materials using method B.

Finally, method F is adopted to determine the bi-modulus of the epoxy resin based on the horizontal on *AB* and vertical displacements on *CF*. Due to the explicit relationship between the elastic modulus and the displacement, the bi-modulus of the epoxy resin can be directly determined according to Eqs. (10) and (11). The estimated bi-moduli of the epoxy resin are shown in Fig. 11.

From Fig. 11, it is found that the average elastic tensile modulus E_t is about 2.59 GPa, the average elastic compressive modulus E_c is about 6.13 GPa. The tensile modulus is also far smaller than the compressive modulus. Comparing with the results determined by method C, this estimation of the E_t and E_c of the epoxy resin are also acceptable. It is indicated that the simplified method F is applicable in some cases if only the horizontal displacement on the diameter *AB* or the vertical displacement on the diameter *CF* is available. However, the simplified method F could significantly overestimate the elastic tensile modulus E_t, and a little underestimate the elastic compressive modulus E_c of materials. Based on the laboratory tests, Liu [4] pointed out that the elastic tensile and compressive modulus of the epoxy resin is 3.767 GPa and 3.329 GPa. This result is different from that estimated by the test theory proposed in this study. The possible reasons are that the E_t and E_c of the epoxy resin are estimated based on the displacement field at the moment when $P/lR=36.1$ MPa in this study, while, it is not clear what the stress status of the samples used by Liu [4] is, the elastic

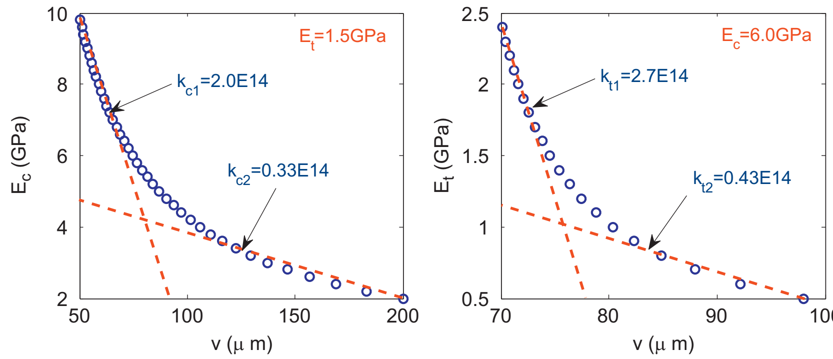

Fig. 9. Sensitivity analyses for the bi-modulus of materials to the measured horizontal displacement on the diameter *AB*.

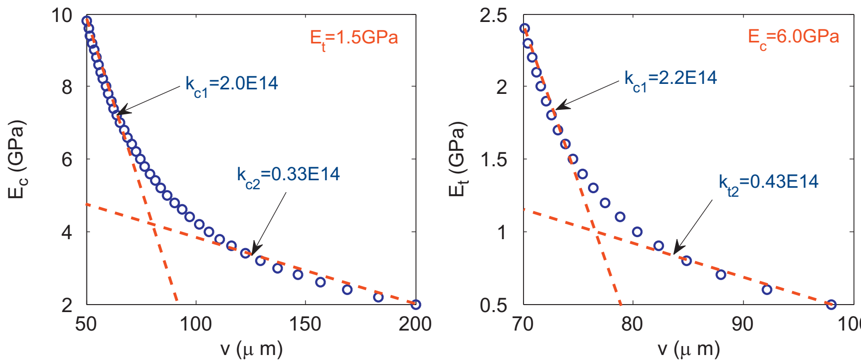

Fig. 10. Sensitivity analyses for the bi-modulus of materials to the measured vertical displacement on the diameter *CF*.

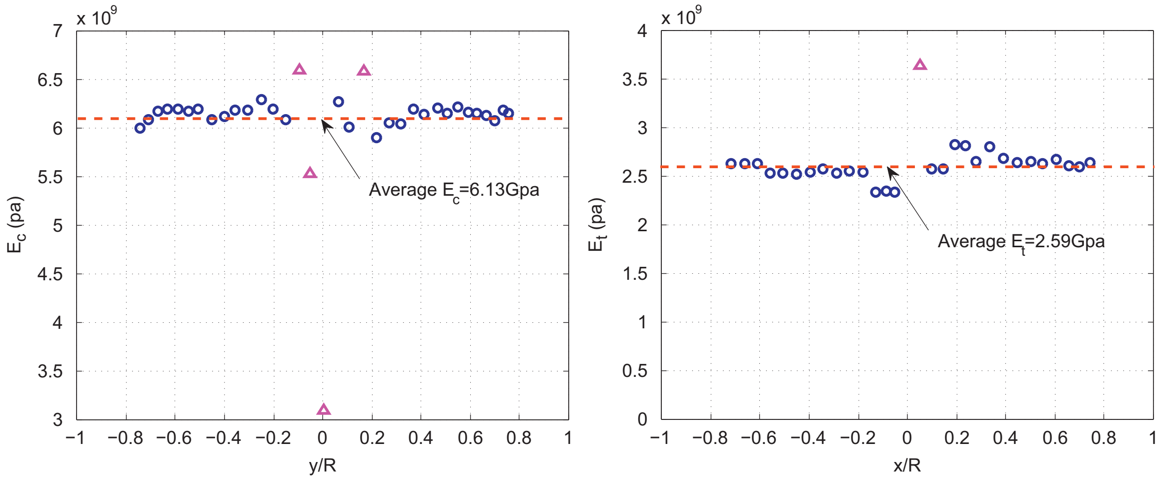

Fig. 11. Bi-modulus of the epoxy resin at the moment when $P/lR = 36.1$ MPa determined by adopting the horizontal on the diameter *AB* plus Eq. (10), and the vertical displacement on the diameter *CF* plus Eq. (11) (method F).

parameters of the epoxy resin may not be kept constant during the loading process. The variation of elastic parameters of rock materials during the loading process will be discussed in the following section.

It also can be seen that the bi-modulus of the epoxy resin determined by method F has good consistency when the point monitored is far away from the central point of disk, for example, $|x/R|$ or $|y/R| > 0.2$. In the zone near to the central point of disk, the results is scattered, thus it is not reliable. The reason would be that the displacement in the zone near to the central point of disk is apparently small, and the measuring accuracy of the DIC technique is not enough to capture the real displacement.

3.2. Gong et al. experiment

Gong et al. [3] performed a series of Brazilian disk tests of rock materials to monitor the displacement of points A, B, C and F. Here, the displacement results of three samples (1-1, 3-1 and 12-1) are cited to validate the proposed test theory. Due to that C and F are the singularity of stress field, the elastic compressive modulus cannot be determined using the displacement of C and F points. Here, only the displacements of points A and B are used to determine the elastic tensile modulus. Due to the fact points A and B are symmetric point, methods A–C are not applicable here. Only the simplified method F can be used to estimate the elastic tensile modulus of rock materials.

In the tests, the applied force P and the displacements of points A and B are recorded during the tests. Fig. 12 shows the recorded results for the three samples. It should be noted that the recorded displacement in Fig. 12 is two times of the real displacement of points A and B due to the test method adopted in [3].

The diameter D of disk is about 50 mm, the ratio between D and thickness l is 1.0. Poisson's ratio is determined as 0.26 through the conventional test method. According to the test theory proposed in this study, the variation processes of the elastic tensile modulus of the three samples are determined as that shown in Fig. 13.

It can be seen from Fig. 13 that the variation range of the elastic tensile modulus E_t is about 0.7–4.0 GPa. In the loading process, the elastic tensile modulus E_t is not a constant, but a variable. The obvious trend is that the elastic tensile modulus increases as the time and loading force increases when the samples are still in the state of elastic deformation under loading. At the beginning stage of loading, the micro fractures in the region near to the diameter CF open under the developed small tensile stress. It results in the horizontal displacement at point A or B which increases relatively quick. The elastic tensile modulus of rock materials seems to be relatively small. Later, the horizontal displacement at point A or B increases at a normal rate, the elastic tensile modulus of the rock materials significantly increases. This variation process of elastic tensile modulus of rock materials is a big challenge to the previous knowledge that thinking the elastic parameters of materials are constants during the elastic deformation process. The phenomenon of the variation of elastic parameters was also observed by Gong et al. [3].

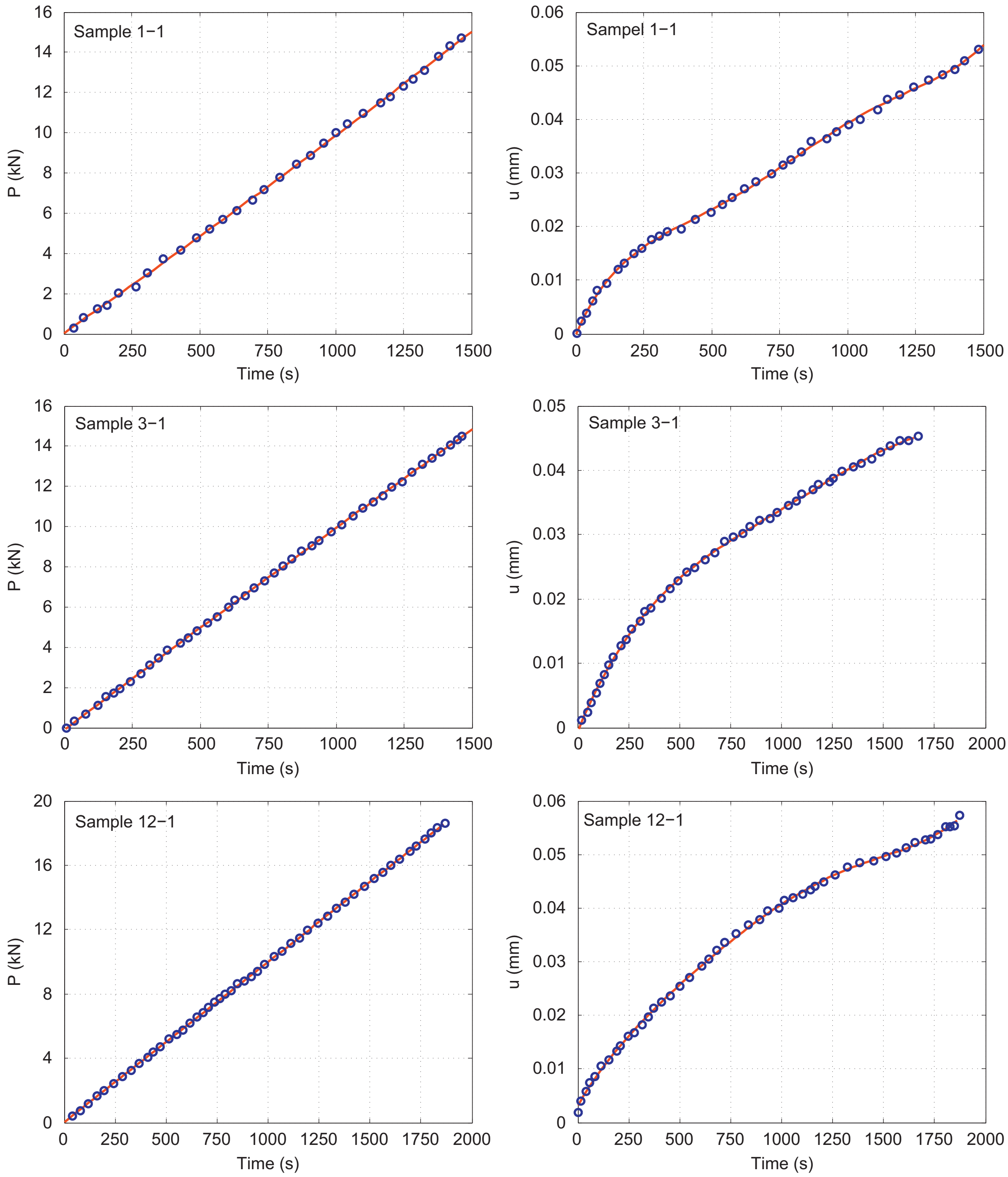

Fig. 12. Recorded force applied on the rock disk and the corresponding displacements at points A and B.

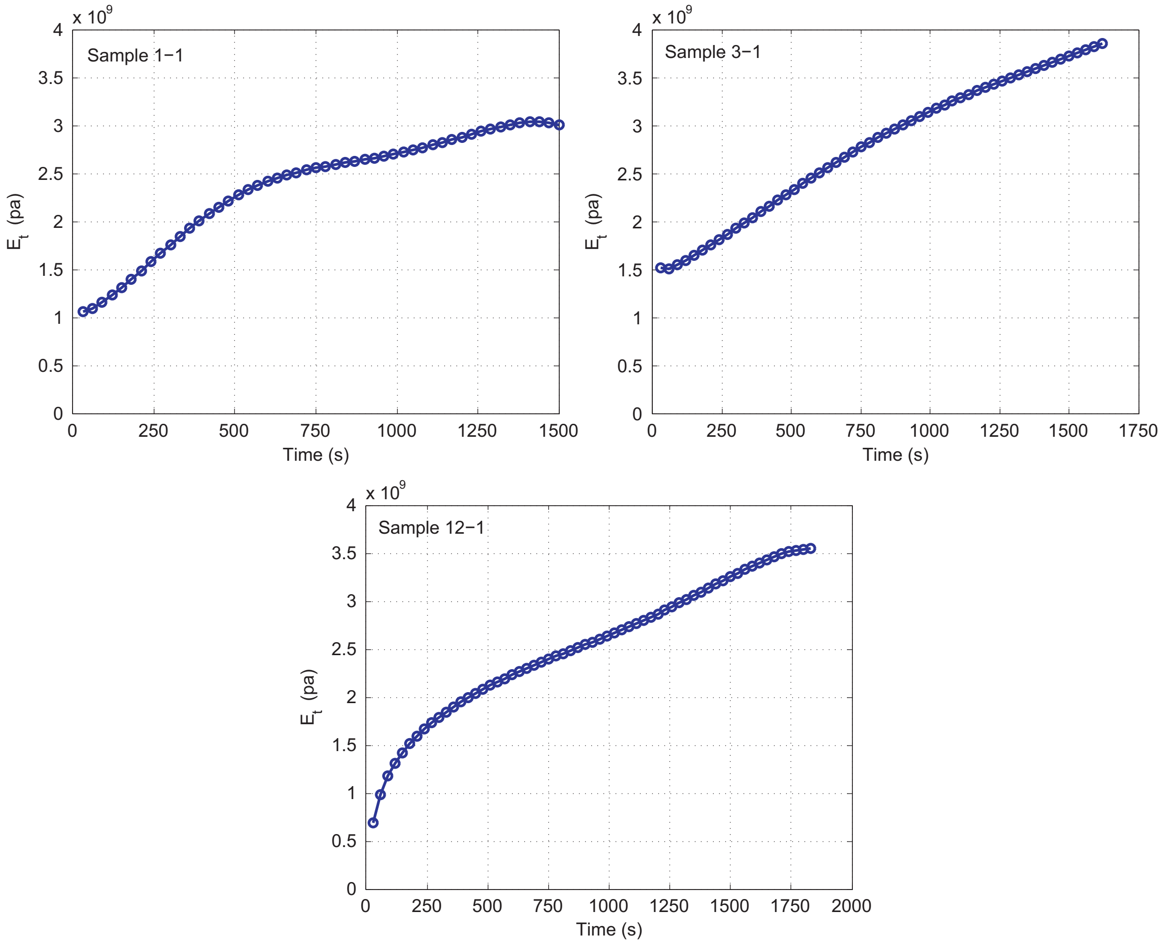

Fig. 13. Variation processes of the elastic tensile modulus E_t of the rock materials determined by method F using the recorded displacement at points A and B.

In laboratory tests, how to determine the elastic tensile modulus E_t of rock materials for engineering design is a problem for engineers. Due to the fact that what we need is the elastic parameter, we should determine the E_t when the disk is being the elastic status. Obviously, it is not reasonable to determine the E_t according to the final failure load and the maximum displacement measured at point A or B. The reason is that some zones in the disk would be in plastic status when the disk is approaching the failure status, and the stresses field based on elastic theory cannot describe the real stresses in the disk. It is also not suggested to estimate the E_t of rock materials based on the loaded force and the displacement measured at the beginning stage of test process. The reason is that the displacement measured at points A and B at the beginning stage contains the contribution of the micro-fractures opening in the region near to the diameter *CF*. In this study, we would suggest that the elastic tensile modulus E_t is determined when the loading force is half of the failure load. However, the national specifications, such as the Chinese national standards for rock mass tests [11], also recommend to determine the elastic parameters of rock materials when the samples are loaded by the half of failure load.

4. Conclusion

In order to consider the differences of tensile and compressive behavior of rock materials, and provide more reliable design elastic parameters for rock engineering, in this study, a completely analytical test theory is developed to determine the elastic tensile modulus E_t and the elastic compressive modulus E_c simultaneously through the deformation and displacement measurement in the Brazilian disk loaded by a pair of radial concentrated forces. Two groups of experimental data of displacements measured from the epoxy resin and the sand rock in the Brazilian disk test are used to validate the feasibility of the proposed test theory. The result shows that the test theory is feasible and convenient to determine the elastic tensile and compressive modulus of rock materials simultaneously.

Based on the proposed test theory, totally four kinds of method are suggested to estimate the E_t and E_c of materials simultaneously. The first method (labelled as method A) only involves Eq. (7) and the horizontal displacement on the diameter *AB*. Due to the high sensitivity of the E_c of materials to the horizontal displacement, it is not recommended to estimate the E_c of materials using method A. The second method (labelled as method B) only involves Eq. (8) and the vertical displacement on the diameter *CF*. Due to the high sensitivity of the E_t of materials to the vertical displacement, it is also not recommended to estimate the E_t of materials using method B. The third method (labelled as method C) needs to use both Eqs. (7) and (8), and both the horizontal and vertical displacement. The priorities of methods A and B are combined in method C. Therefore, the estimated E_t and E_c of materials using method C have the best reliability among these methods. If the simplification that ignoring the difference of E_t and E_c of materials in Eqs. (7) and (8), then the E_t and E_c of materials can approximately be explicitly expressed by Eqs. (10) and (11) (labelled as method F). However, method F would significantly overestimates the E_t of materials, and a little underestimates the E_c of materials. If method F is used to determine the bi-modulus of materials, the displacement data measured at the points near to the central point of disk is not suggested to be used.

The results of E_t and E_c indicate that the elastic tensile modulus is generally less than the elastic compressive modulus of materials. The error of displacement measurement in tests results in the probabilistic distribution of the estimated E_t and E_c. Through the fitting analysis, it is found that the estimated E_t and E_c follow the t location-scale distribution very well.

The elastic tensile modulus determined by the recorded displacement is a variable, rather than a constant in the loading process. This is a big challenge to the previous knowledge that thinking the elastic parameters of materials are constants during the elastic deformation process.

Acknowledgment

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